9/13/21

MATH 323-01

12.6 - Quadric Surfaces

Ex: Understand the surface w/ equation

$$x^2 - y^2 - z^2 - 4x - 2z + 3 = 0.$$

$(a+b)^2$
$= a^2 + 2ab + b^2$

<u>Sol</u>: First rewrite the equation (via completing the square):

$$0 = x^2 - y^2 - z^2 - 4x - 2z + 3$$

if and only if

$$0 = (x^2 - 2\cdot 2x + 2^2 - 2^2) + (-y^2) - (z^2 + 2\cdot 1z + 1^2 - 1^2) + 3$$

iff $0 = (x-2)^2 - 2^2 - y^2 - (z+1)^2 - (-1^2) + 3$

iff $0 = (x-2)^2 - y^2 - (z+1)^2$

Now, let's understand the cross-sections of this picture with respect to (wrt) the coordinate plane (shifted).

when $z = k$: ← Konstant

$$0 = (x-2)^2 - y^2 - (k+1)^2$$

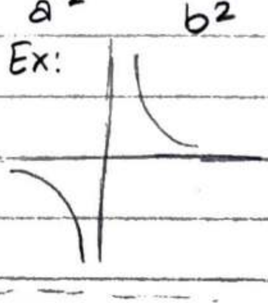
i.e. $(x-2)^2 - y^2 = (k+1)^2$ ← Hyperbola

<u>Conic Sections</u>

Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = c$

Ex:



Parabola: $\frac{x^2}{a^2} - \frac{y}{b} = c$

when $y = k$:

$0 = (x-2)^2 - k^2 - (z+1)^2$

i.e. $(x-2)^2 - (z+1)^2 = k^2$

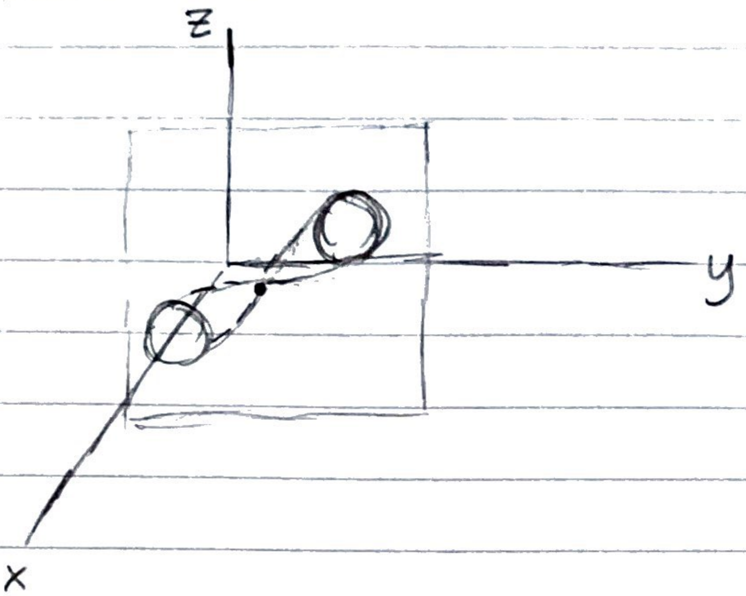
Hyperbola

when $x = k$:

$0 = (k-2)^2 - y^2 - (z+1)^2$

i.e. $y^2 + (z+1)^2 = (k-2)^2$ ← Ellipse (circle or point at $k=2$)

So a picture:



Cone:

$$\frac{-x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$$

Equation	Name	$x=k$	$y=k$	$z=k$
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Ellipsoid	Ellipse (general)	Ellipse (general)	Ellipse (general)
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z}{c}=0$	Elliptic Paraboloid	Parabola	Parabola	Ellipse
$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z}{c}=0$	Hyperbolic Paraboloid	Parabola	Parabola	Hyperbola
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	One-sheet Hyperboloid	Hyperbola	Hyperbola	Ellipse
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$	Cone	Hyperbola	Hyperbola	Ellipse (or point)
$\frac{-x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Two-sheet Hyperboloid	Hyperbola	Hyperbola	Ellipse (or nothing)

13.1 - Space Curves

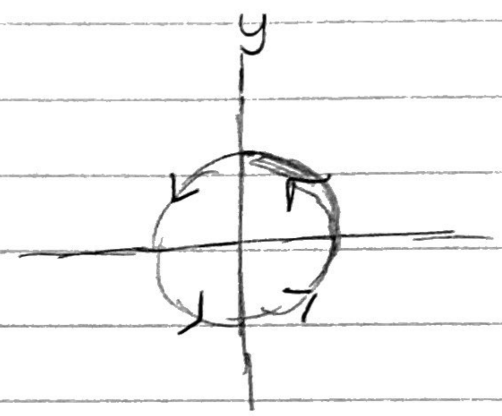
- A space curve is a function:

$$\vec{r} : I \to \mathbb{R}^n$$

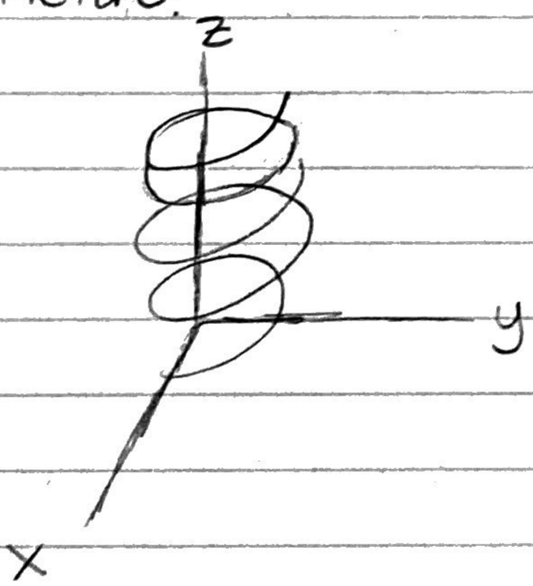
Ex: The helix is the space curve

$$\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle.$$

Shadow in xy-plane:



Picture:



Defn: The limit of a space curve

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ at "time" $t = a$ is the componentwise limit if they all exist. I.e.

$$\lim_{t \to a} \vec{r}(t) = \lim_{t \to a} \langle x(t), y(t), z(t) \rangle$$

$$= \left\langle \lim_{t \to a} x(t), \lim_{t \to a} y(t), \lim_{t \to a} z(t) \right\rangle$$

Exercise: Compute the limit of

$\vec{r}(t) = \langle (4+\sin(20t)\cos(t), (4+\sin(20t)\sin(t),$
$\cos(20t) \rangle$
at $t = \frac{6\pi}{8}$